

# Incentivizing Hosts via Multilateral Cooperation in User-Provided Networks: A Fluid Shapley Value Approach

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## ABSTRACT

Successful operation of User-Provided Networks (UPN) requires that both of Internet Service Provider (ISP) and self network-operating users (hosts) cooperate appropriately in terms of resource sharing and pricing strategy since ISP and hosts have a multilateral reliance on each other with respect to virtual infrastructure expansion and Internet connectivity. However, it has been underexplored whether such cooperation provides sufficient incentive to ISP and hosts under a setup where ISP and hosts are fully included, having a high dependence on how to cooperate and how to distribute the resulting cooperation worth. In this paper, we model a market of UPN, consisting of ISP, hosts, and clients via game theory, where we model various heterogeneities in terms of (i) willingness to pay and mobility pattern of clients, (ii) hosts' QoS, and (iii) type of cooperation among ISP and hosts. The key technical challenges lie in the natural mixture of cooperative and non-cooperative game theoretic angles, where the worth function—one of the crucial components in coalitional game theory—comes from the equilibrium of an embedded, non-cooperative two-stage dynamic game. We consider the Shapley value as a mechanism of revenue sharing and overcome its hardness in characterization by taking the fluid limit when the number of hosts and clients is large. Our analytical studies reveal useful implications that in UPN when and how much economic benefits can be given to the players and when they maintain their grand coalition under what conditions, referred to as stability.

## CCS CONCEPTS

• Networks → Network economics; • Theory of computation → Solution concepts in game theory;

## KEYWORDS

Network Economics, Game Theory, User-Provided Network

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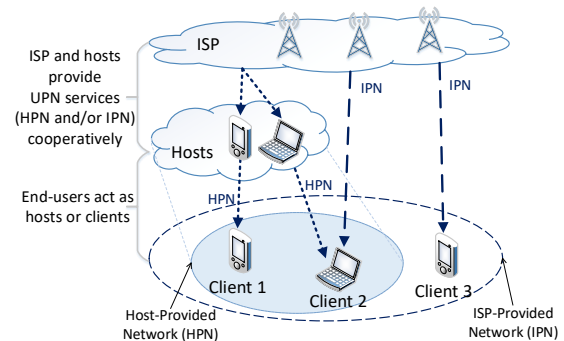


Figure 1: Structure of network-assisted UPN. ISP/hosts cooperatively provide HPN and/or IPN, subscribed by clients.

## 1 INTRODUCTION

Internet traffic is rapidly increasing, where the exponential growth of mobile traffic is prominent [7]. As a cure for such ever-increasing demands on mobile data traffic, User-Provided Network (UPN) services that enable an end-user to behave as a micro Internet operator (called host) by relaying data traffic to a data consuming user (called client), have been significantly grown. For example, Open Garden [22] provides a software platform in which end-users form an autonomous mesh network for exchanging data traffic. FON [9] and Karma [13] also provide platforms for network-assisted UPN services, where fixed/mobile hosts help to increase ISPs' connectivity by sharing their fixed WiFi APs/mobile hotspots to other data consuming clients.

The main interest of this paper lies in such network-assisted UPN services illustrated in Figure 1. Successfully operating UPNs requires that both of Internet Service Provider (ISP) and hosts cooperate appropriately in terms of resource sharing and pricing strategy for clients, since ISP and hosts have multilateral reliance with respect to virtual infrastructure expansion and Internet connectivity. However, it has been underexplored whether such a cooperation provides sufficient incentive to ISP and hosts under a setup where ISP and hosts are fully included, having high dependence on how to cooperate and how to distribute the resulting cooperation worth.

It seems typical for ISPs to make contract in the form a bilateral settlement with each host for incentivizing her cooperative resource sharing. However, bilateral settlement may form locally optimal connectivities between ISP and hosts, naturally restricting the power of cooperation and limiting hosts' incentives to share their resources. The lack of enough consideration of incentivizing hosts may cause undesirable settlement such as to break the global connectivity as well as to degrade the performance of UPN services. For example, AT&T has blocked the access to Google Play through Open Garden [28], and many ISPs including AT&T and Verizon have paid additional charges for tethering [23]. Only maximizing

the individual profit often discourages users to act as UPN hosts, thereby hindering the growth of UPN and ultimately leading to resource wastage.

In this paper, we model a market of UPN, consisting of ISP, hosts, and clients via a mixture of cooperative and non-cooperative game theoretic approaches. We study the impact of the cooperation between ISP and hosts on the non-cooperative interaction of clients' response to the cooperation, where we aim at answering the following two key questions in diverse dimensions: (i) How much multilateral cooperation among ISP and hosts affects the economic benefits of ISP, hosts, and clients and (ii) under what conditions the cooperation of ISP and hosts would be sustained. We summarize our main contributions in what follows:

- (a) First, we quantify the cooperation effect as a form of the total revenue (often called the worth function in coalitional game theory) for any possible cooperation among ISP and hosts. Since strategic clients are involved in our modeling, the total revenue via cooperation comes from the result of how cooperating ISP/hosts and clients behave in the market, thus requiring a non-cooperative game theoretic approach. We believe that it is one of the unique and analytically challenging features in our model that coalitional game plays a role of a key framework of our analysis, but that a non-cooperative game is also embedded as a module to understand the relation between a cooperation of ISP/host and clients. Besides, towards more practicability we model various heterogeneities in (i) willingness to pay and mobility pattern of clients, (ii) hosts' QoS, and (iii) cooperation types among ISP and hosts.
- (b) Second, we study the impact of the different cooperation types, for which we consider two cases (i) when ISP and hosts share their resource as well as the price decisions (full cooperation) and (ii) when they only share their resources (partial cooperation) and leave the pricing decision as a private information. This is motivated by the fact that ISP and hosts may not decide on their prices together and even want to hide them in order to reduce a cost for security and information exchange. Clearly, depending on how strongly ISP and hosts cooperate, all other economic measures and the degree of willingness to cooperate would differ.
- (c) Third, we consider the famous Shapley value (SV) [24] as a rule of sharing the total revenue. As is well known, SV is a kind of fair-share mechanism that satisfies good axioms (efficiency, fairness, symmetry) and derives the final revenue share distributed to each of ISP and hosts, as widely used to analyze the gains from cooperative behaviors in communication networks, e.g., peer-assisted services [5, 21], cooperation of ISPs [16, 18, 19]. However, numerically computing the SV is generally hard (i.e., NP-hard), and analytically characterizing it is even more challenging. As an approximation, we take the fluid limit of the SV when the number of hosts and clients scales called *fluid Shapley value* under mild conditions. This allows the analytical characterization of the revenue distribution among ISP and hosts, which in turn enables us to study when they maintain their grand coalition under what conditions, referred to as stability.

## 1.1 Related Work

The proliferation and commercialization of UPN is due to the recent interests in actively using the resources of edge devices, e.g., edge/fog computing [3, 6, 8] and crowdsourced mobile streaming [27]. UPNs are broadly classified into network-assisted UPNs (e.g., Karma [13]) and autonomous UPNs (e.g., Open Garden [22]), depending on how end-users' resources are shared (see a nice survey in [11]). In brief, in network-assisted UPNs, which is also our focus, the ISP (i.e., Mobile Network Operator (MNO) or Mobile Virtual Network Operator (MVNO)) enables its subscribers to operate as fixed/mobile hosts (typically serving WiFi) and provides Internet connectivity for others, whereas autonomous UPNs allow mobile users to create a mesh network and share their Internet connections. Another criterion of looking at UPNs is based on how players make contracts, which define a mechanism of offering the incoming revenue to each player: bilateral and multilateral settlement.

The related work on *network-assisted* UPNs with *bilateral* settlement includes [1, 10, 14, 17, 20, 29]. The authors in [10] studied an MVNO's optimal reimbursement policy to hosts for encouraging their resource sharing, so as to maximize the MVNO's profit. In [29], an optimal tethering pricing is considered where there are two competitive or cooperative MNOs, showing that the optimal tethering prices become zero but that revenues of MNOs and the utility of hosts increase under cooperative MNOs due to the increase of available resources. In [1], the authors investigated the optimal pricing policy of ISP for clients to maximize the provider's profit. In [20], the authors studied the optimal pricing of competitive access service operators, which are licensed band operators or social community operators. In [17], the authors analyze user behavior and the pricing of a provider operating a crowdsourced wireless community network. In [14], a market of ISP and users is considered, where each user can choose to be either of host or client, with each offered a difference price. All of these works largely differ from ours in that they use bilateral settlements, so that only a pairwise pricing/relation is studied between ISP and host, ISP and client, or host and client.

The papers in [12, 26] consider multilateral settlements in *autonomous* UPNs. The authors in [12] consider a market with ISP and users (who can be either of a host or a client), where they study an incentive mechanism and a pricing rule made by the NBS (Nash Bargaining Solution)-based resource sharing rule, considering an autonomous UPN, differing from our coalitional game theoretic framework. The work in [26] is close to ours in terms of an analytical tool of coalitional game theory. However, since autonomous UPNs are their main interest, the key difference is that they focus on the cooperation only among users, but we focus on the cooperation among ISP and hosts. To the best of our knowledge, this paper is the first that studies the cooperation among "providers" (i.e., ISP and hosts) in network-assisted UPNs. Additionally, our analytic framework consists of a complex inter-play between coalitional game theory and non-cooperative game theory, where our theoretical quantification of coalition worth and (fluid) Shapley value will be of broad, intellectual interest to other areas, e.g., smart grid.

## 2 MODEL

### 2.1 System Model

**Internet Service Provider (ISP).** We consider a single Internet service provider (ISP)  $I$  which provides the Internet access service with the global coverage of the whole region. It owns wired backbones, backhaul networks, and cellular and/or WiFi access networks, which we call ISP-provided network (IPN). Such an ownership may be actual or virtual, and thus it can be either MNO (e.g., AT&T, Sprint, and Cingular) or MVNO (e.g., Karma, Project Fi, and TracFone).

**Hosts.** There is a set  $\mathcal{H} = \{1, 2, \dots, H\}$  of  $H$  host users (or simply hosts throughout this paper) spread in the entire region. Each host is equipped with wired or wireless Internet connectivity through ISP  $I$ , and thus is able to operate a self-constructed wireless host-provided network (HPN) with its own local coverage by relaying uplink/downlink traffic of end-users (or also called *clients* throughout this paper). We assume that the regions covered by hosts are disjoint, thus we henceforth use  $h \in \mathcal{H}$  to index a particular host. For simplicity, we assume that the union of hosts' coverage is equal to the entire region.

**Clients.** There are  $m$  clients who correspond to data-consuming end-users through IPN and/or HPN. Each client belongs to a class in the set of mobility classes  $\mathcal{D} = \{1, 2, \dots, D\}$ , where there are  $m^d$  clients in each class  $d$ , having different mobility patterns, i.e.,  $m = \sum_{d \in \mathcal{D}} m^d$ . To model a mobility pattern in class  $d$ , denote  $[\Pi^d(h)]_{h \in \mathcal{H}}$  be the  $H$ -dimensional vector, where  $\Pi^d(h)$  is the probability (or long-term time portion) that a class- $d$  client stays at the host  $h$ 's region.

**Service.** Each client can subscribe to one of three services: (i) HPN-only, (ii) IPN-only or (iii) HPN+IPN. As the name implies, in HPN-only and IPN-only, a client has to access only HPNs and only IPN for the Internet connection, respectively, whereas in HPN+IPN, she can use both HPN and IPN. A client would choose a service in  $\mathcal{S} \triangleq \{n, h, i, w\}$ , where we use the 'n', 'h', 'i' and 'w' to refer a client's selection of subscribing to no-Internet access, HPN-only, IPN-only and HPN+IPN, respectively. Thus, a client pays no money if she chooses no-Internet. Otherwise, for clients of class  $d$ , ISP and hosts set the service prices  $p_h^d, p_i^d$  and  $p_w^d$ , which is a kind of class-dependent pricing. Denote the price vector  $\mathbf{p}^d = (p_h^d, p_i^d, p_w^d)$ .

### 2.2 Cooperation among ISP and Hosts, and QoS

**Coalition.** The ISP  $I$  and the hosts in  $\mathcal{H}$  cooperate by sharing their resources with the following motivation: ISP can enhance the QoS provided for its subscribers due to the virtual expansion of its infrastructure, and a host can share its underutilized resource to make some revenue. Under a cooperation, ISP helps hosts' Internet connection service by supporting backbone connectivity and hosts share their HPNs and contribute to ISP's QoS enhancement by supporting HPN+IPN service, as described in Figure 2(a). Consider a coalition  $C = \{I\} \cup \mathcal{H}'$  that consists of the ISP and a set of hosts  $\mathcal{H}' \subset \mathcal{H}$ , in which case only the hosts in  $\mathcal{H}'$  are able to provide the global Internet connection to clients. Of particular interest is the grand coalition  $\mathcal{G} = \{I\} \cup \mathcal{H}$  that contains all hosts. Note that we

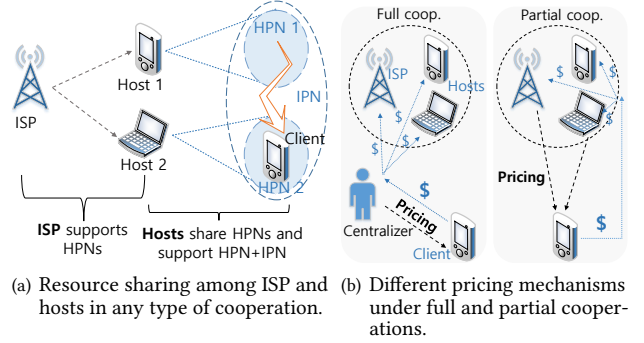


Figure 2: Cooperation among ISP and hosts.

only consider the coalitions that always contain  $I$ , because without it, HPNs are incapable of providing the Internet service.<sup>1</sup>

One of our key interests is the economic gain due to the cooperation among the players in a coalition, often referred to as (*coalition*) *worth*. The worth from a coalition  $C$  depends on how the players in  $C$  cooperate. To study it, we consider the following two cooperation types, which differ in terms of its degree and structure of cooperation as illustrated in Figure 2(b) (see Section 4 for more mathematical treatments of these cooperation types).

- **Full cooperation:** ISP and hosts behave just like a single organization in the sense that there exists an omniscient centralizer supervising the decision of the prices  $p_h^d, p_i^d$  and  $p_w^d$  to maximize its total worth, and the worth is shared a posteriori according to some agreed rule.
- **Partial cooperation:** ISP and hosts cooperatively operate the Internet service just like in the full cooperation by sharing their resources, but choose the service prices in a non-cooperative manner. In other words, the prices  $p_i^d$  and  $p_w^d$  are determined by ISP, whereas each host  $h$  sets the price  $p_h^d$ . Then, similarly to the full cooperation, they share the total revenue a posteriori in agreement with a given rule.

**Network QoS.** We assume that ISP provides higher average QoS than hosts. One may think that it is often the case that a user experiences higher bandwidth when it connects to a nearby WiFi AP than LTE. However, our focus is on the long-term average QoS that abstracts all features of a network service such as bandwidth, reliability, security, seamless handoffs, etc<sup>2</sup>. Let the IPN's QoS be  $q > 0$ , and for each HPN  $j$ , let  $k_h(j)q$  and  $k_w(j)q$  be the QoS of HPN-only and HPN+IPN, where  $k_h(j) < 1$  and  $k_w(j) \in [1, 1 + k_h(j)]$  (where we reflect the effect of utilizing both HPN and IPN in  $k_w(j)$ ).

Considering clients' mobility, the expected QoS of clients of class  $d$  in the coalition  $C$  are respectively given by:

$$\begin{aligned} \text{HPN-only : } k_h^d q &= \sum_{j \in C \setminus \{I\}} \Pi^d(j) k_h(j) q, \\ \text{HPN+IPN : } k_w^d q &= \sum_{j \in C \setminus \{I\}} \Pi^d(j) k_w(j) q + \sum_{j' \in \mathcal{G} \setminus C} \Pi^d(j') q, \quad (1) \end{aligned}$$

where recall  $\mathcal{G}$  denotes the grand coalition. Note that IPN-only service provides the QoS-level of  $q$ , and the QoS of HPN-only service is given by the sum of expected QoS from all HPNs in the coalition

<sup>1</sup>In coalitional game theory, ISP  $I$  is called a veto player.

<sup>2</sup>However, even the opposite case can be similarly analyzed by applying the mathematical techniques developed in this paper.

$C$ . Thus, the first and the second terms of HPN+IPN service QoS correspond to the expected HPN+IPN QoS from the all HPNs in the coalition  $C$  and the expected IPN-only QoS from the all HPNs in  $\mathcal{G} \setminus C$ , respectively.

### 3 GAME FORMULATION AND FLUID SHAPLEY VALUE

In this section, we introduce our game model for user-provided networks to capture not only the impact of cooperation effect, but also that of the competition between a given coalition of ISP/hosts and clients. To this end, we use a model that first plays a two-stage dynamic game between ISP/hosts and clients, producing the cooperation's worth, from which a coalitional game is played inside the cooperation of ISP and hosts.

#### 3.1 Coalitional Game and Revenue Sharing

**Coalitional game and worth.** We denote by  $(\mathcal{G}, v)$  a coalitional game, where  $\mathcal{G}$  is a set of all players, i.e.,  $\mathcal{G} = \mathcal{H} \cup \{I\}$ . Then,  $v : 2^{|\mathcal{G}|} \mapsto \mathbb{R}$  is a worth function that defines the worth of every coalition  $C \subseteq \mathcal{G}$ , which is the total revenue earned by the non-cooperative interaction with the clients subscribing the ISP and the hosts in  $C$ , as illustrated in Figure 3. We henceforth describe our game for a given coalition  $C$ , thus all the notations such as revenue, price, QoS, and clients' utility depends on  $C$ . However, for notational simplicity, we omit their dependence on  $C$ , unless explicitly needed.

Let  $\pi_h^d(\mathbf{p}^d)$ ,  $\pi_i^d(\mathbf{p}^d)$ , and  $\pi_w^d(\mathbf{p}^d)$  be the revenues from class- $d$  clients subscribing to HPN-only, IPN-only, and HPN+IPN, given a price vector  $\mathbf{p}^d = (p_h^d, p_i^d, p_w^d)$ . Then, the worth  $v(C)$  in coalition  $C$  is given by:

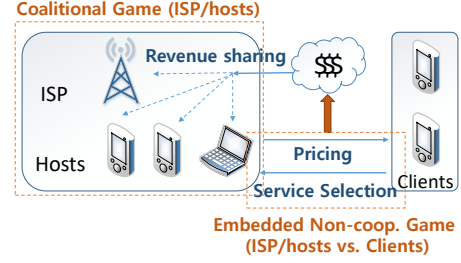
$$v(C) = \sum_{d \in \mathcal{D}} \pi_h^d(\mathbf{p}^d) + \sum_{d \in \mathcal{D}} \left( \pi_i^d(\mathbf{p}^d) + \pi_w^d(\mathbf{p}^d) \right), \quad (2)$$

where the first and the second terms correspond to the revenues of the hosts in coalition  $C$  and ISP, respectively. Note that hosts not in the coalition  $C$  earn no revenue.

As will be described in Section 3.2, the *equilibrium* price vector  $\mathbf{p}^d = (p_h^d, p_i^d, p_w^d)$  comes from an *embedded* non-cooperative, two-stage dynamic game between the coalition of ISP/hosts and clients, which in turn depends on the type of cooperation (i.e., full or partial). We will also present the exact form of the revenue functions  $\pi_h^d(\mathbf{p}^d)$ ,  $\pi_i^d(\mathbf{p}^d)$  and  $\pi_w^d(\mathbf{p}^d)$  in (5) of Section 3.2.

**Revenue sharing.** When the worth of a coalition  $C$  is obtained, a certain rule of sharing the worth determines the amount of individual revenue distributed to each player in  $C$ . We consider a famous rule, called *Shapley value*, which produces a unique revenue allocation, satisfying the axioms such as (i) efficiency, (ii) symmetry and (iii) fairness/balanced contribution<sup>3</sup>. Due to space limitation, we refer the readers to [24] for more detail. In a coalition  $C$  with the worth function  $v$ , Shapley value determines each player  $j$ 's share

<sup>3</sup>Strictly speaking, in coalitional game theory we call the value satisfying these axioms Shapley value only for the grand coalition, and for a sub-coalition, we call it Aumann Dr eze value [2]. However, we just use the term 'Shapley value' throughout this paper for simplicity.



**Figure 3: UPN game structure. The worth of coalition is determined by the embedded non-cooperative game between the coalition players (ISP/hosts) and clients.**

$\varphi^j(C, v)$  based on her average marginal contribution, formally defined as: for each player  $j$ ,

$$\varphi^j(C, v) = \sum_{\mathcal{E} \subseteq C \setminus \{j\}} \frac{|\mathcal{E}|!(|C| - |\mathcal{E}| - 1)!}{|C|!} (v(\mathcal{E} \cup \{j\}) - v(\mathcal{E})), \quad (3)$$

where  $v(\mathcal{E} \cup \{j\}) - v(\mathcal{E})$  is a marginal contribution of  $j$  on set  $\mathcal{E}$ . For notational simplicity,  $\varphi^j(C)$  is used interchangeably with  $\varphi^j(C, v)$ .

#### 3.2 Embedded Non-cooperative Dynamic Game

We now describe a two-stage dynamic game, given a coalition  $C$ , that leads to the worth function  $v(C)$ , by deciding  $\mathbf{p}^d = (p_h^d, p_i^d, p_w^d)$  and thus the total revenues of three services. This two-stage dynamic game is the one between ISP/hosts coalition and clients, where the ISP and the hosts in  $C$  choose the service prices and each client, as the price-taker, chooses the service to subscribe.

**Utility of clients.** We first introduce a client's (in a mobility class  $d$ ) utility function  $u^d : \mathcal{S} \times \mathbb{R}^3 \mapsto \mathbb{R}$  with her strategy being the service to subscribe:

$$u^d(s, \mathbf{p}^d; \theta) = \begin{cases} 0, & \text{if } s = 'n', \\ \theta k_h^d q - p_h^d, & \text{if } s = 'h', \\ \theta q - p_i^d, & \text{if } s = 'i', \\ \theta k_w^d q - p_w^d, & \text{if } s = 'w', \end{cases} \quad (4)$$

where  $k_h^d$  and  $k_w^d$  are as in (1). To model clients' heterogeneity, they are assumed to have different willingness to pay  $\theta$ , where as popularly modeled in e.g., [4],  $\theta$  is a uniformly random value over the interval  $[0, 1]$ . It means that a client with a larger  $\theta$  prefers a higher QoS than the one with a smaller  $\theta$ .

**Revenues of ISP and hosts.** Again, given a coalition  $C$ , the strategy of  $C$  is the price vector of all three services,  $\mathbf{p} = [p^d]_{d \in \mathcal{D}}$ . Note that it can be chosen by a single centralized coordinator (in full cooperation), or by each of ISP and hosts independently (in partial cooperation). Due to clients' difference in willingness to pay, given  $\mathbf{p}^d$ , four difference continuous intervals  $\Theta_s^d(\mathbf{p}^d) \subseteq [0, 1]$ , for all  $s \in \mathcal{S} = \{n, h, i, w\}$  are defined where each interval  $\Theta_s^d(\mathbf{p}^d)$  corresponds to the market share of a service  $s$  for class- $d \in \mathcal{D}$  clients (i.e., portion of class- $d$  clients subscribing to service  $s$ ). Then, the revenue of the service  $s \in \mathcal{S} \setminus \{n\}$  from class- $d \in \mathcal{D}$  clients is given by:

$$\pi_s^d(\mathbf{p}^d) = p_s^d \cdot \left( m^d(C) \cdot \int_0^1 \mathbf{1}_{\{\theta \in \Theta_s^d(\mathbf{p}^d)\}} d\theta \right), \quad \forall s \in \mathcal{S} \setminus \{n\}, \quad (5)$$

where we denote by  $m^d(C)$  the number of clients covered by ISP and hosts in the coalition  $C$ . Then, the total aggregate revenue

over the mobility classes in  $\mathcal{D}$  is simply  $\sum_{d \in \mathcal{D}} \pi_s^d(\mathbf{p}^d)$ . We now describe more detailed description of our embedded dynamic game.

#### Embedded Two-stage Dynamic Game for a given coalition $C$

**Stage I: Pricing of cooperation.** ISP and hosts decide their prices  $\mathbf{p}^d = (p_h^d, p_i^d, p_w^d)$  for three services  $\{h, i, w\}$  over all classes  $d \in \mathcal{D}$ .

- **Full cooperation:** A virtual centralizer decides the prices to maximize the worth of cooperation  $C$ :

$$\begin{aligned} \text{Centralizer: } \mathbf{p}^* &= \arg \max_{\mathbf{p}} v(C), \\ &= \arg \max_{\mathbf{p}} \sum_{d \in \mathcal{D}} \pi_i^d(\mathbf{p}^d) + \pi_w^d(\mathbf{p}^d) + \pi_h^d(\mathbf{p}^d) \end{aligned} \quad (6)$$

- **Partial cooperation:** ISP and hosts do not share the pricing strategy, where we assume that ISP is an incumbent of the market and hosts are the entrants, so they play a sequential game. Thus, ISP first sets the prices  $\mathbf{p}_i = [p_i^d]_{d \in \mathcal{D}}$  and  $\mathbf{p}_w = [p_w^d]_{d \in \mathcal{D}}$  and the hosts set the price  $\mathbf{p}_h = [p_h^d]_{d \in \mathcal{D}}$ , to maximize each of the following:

$$\begin{aligned} (i) \text{ ISP: } (\mathbf{p}_i^*, \mathbf{p}_w^*) &= \arg \max_{(\mathbf{p}_i, \mathbf{p}_w)} \sum_{d \in \mathcal{D}} \pi_i^d(\mathbf{p}^d) + \pi_w^d(\mathbf{p}^d), \\ (ii) \text{ Hosts: } \mathbf{p}_h^* &= \arg \max_{\mathbf{p}_h} \sum_{d \in \mathcal{D}} \pi_h^d(\mathbf{p}^d). \end{aligned} \quad (7)$$

**Stage II: Clients' service selection.** Each client in mobility class  $d$  of willingness-to-pay  $\theta$  chooses one of the services among  $\{n, h, i, w\}$ , which correspond to no-internet access, HPN-only, IPN-only and HPN+IPN, so as to maximize her utility:

$$\text{Client: } s^{d,*}(\theta) = \arg \max_{s \in S} u^d(s, \mathbf{p}^d), \quad (8)$$

where  $\mathbf{p}^d$  is given by the ISP/hosts cooperation in Stage I.

## 4 UPN MARKET ANALYSIS: FULL AND PARTIAL COOPERATION

In this section, we first study the equilibrium analysis for a two-stage dynamic game between a given ISP/host coalition  $C$  and clients, under both partial and full cooperations. Our analysis gives us the equilibrium price as well as the per-service total revenue, which in turn produces the form of the worth function  $v(C)$ .

### 4.1 Equilibrium Analysis in Stages I and II

A standard tool of computing the equilibrium is the backward induction, for which we first describe how clients select their services in Stage II for a given price vector  $\mathbf{p} = [p^d]_{d \in \mathcal{D}}$ , followed by the selection of the price vectors by  $C$  at the equilibrium.

**Clients' service selection in Stage II.** Following (4) and (8), it is easy to see that a client in mobility class  $d \in \mathcal{D}$  chooses the strategy  $s^{d,*}(\theta)$  to maximize her utility as follows:

$$s^{d,*}(\theta) = \begin{cases} w, & \text{if } \theta > \frac{p_w^d - p_i^d}{(k_w^d - 1)q}, \\ i, & \text{if } \frac{p_i^d - p_h^d}{(1 - k_h^d)q} < \theta \leq \frac{p_w^d - p_i^d}{(k_w^d - 1)q}, \\ h, & \text{if } \frac{p_h^d}{k_h^d q} < \theta \leq \frac{p_i^d - p_h^d}{(1 - k_h^d)q}, \\ n, & \text{otherwise.} \end{cases} \quad (9)$$

Note that the result in (9) leads to the market share of each service. Thus, the market shares of HPN-only, IPN-only, and HPN+IPN are

$\Theta_h^d = \left[ \frac{p_h^d}{k_h^d q}, \frac{p_i^d - p_h^d}{(1 - k_h^d)q} \right]$ ,  $\Theta_i^d = \left[ \frac{p_i^d - p_h^d}{(1 - k_h^d)q}, \frac{p_w^d - p_i^d}{(k_w^d - 1)q} \right]$ ,  $\Theta_w^d = \left[ \frac{p_w^d - p_i^d}{(k_w^d - 1)q}, 1 \right]$ , respectively. This implies, as expected, Internet access service who offers a higher QoS can attract more clients with larger willingness to pay, and we can have similar interpretations for other services.

**Coalition's price selection in Stage I.** For notational convenience, we use the notations  $v_f(C)$  and  $v_p(C)$  to mean the worth functions for full and partial cooperations, respectively. Theorem 4.1 states the per-service price vectors, total revenues, and the resulting worth functions at the equilibrium.

**THEOREM 4.1.** *Consider a non-empty coalition  $C$  that includes  $I$ . Then, we have the following results at the equilibrium under full and partial cooperations.*

- (i) **Full cooperation.** *The equilibrium service prices for class- $d$  clients are uniquely determined as:*

$$p_h^d = \frac{k_h^d q}{2}, \quad p_i^d = \frac{q}{2}, \quad p_w^d = \frac{k_w^d q}{2} \quad (10)$$

Then, the per-service revenues from class- $d$  clients are:

$$\pi_h^d = \pi_i^d = 0, \quad \pi_w^d = \frac{q k_w^d m^d \sum_{h \in C} \Pi^d(h)}{4}, \quad (11)$$

which leads to the coalition worth  $v_f(C)$ :

$$v_f(C) = \sum_{d \in \mathcal{D}} \frac{q k_w^d m^d \sum_{h \in C} \Pi^d(h)}{4}. \quad (12)$$

- (ii) **Partial cooperation.** *The equilibrium service prices for class- $d$  clients are uniquely determined as:*

$$\begin{aligned} p_h^d &= \frac{k_h^d (1 - k_h^d) q}{2(2 - k_h^d)}, \quad p_i^d = \frac{(1 - k_h^d) q}{2 - k_h^d}, \\ p_w^d &= \frac{(k_w^d - 1) q}{2} + \frac{(1 - k_h^d) q}{2 - k_h^d}. \end{aligned}$$

Then, the per-service revenues from class- $d$  clients are:

$$\pi_h^d = m^d \sum_{h \in C} \Pi^d(h) \frac{q k_h^d (1 - k_h^d)}{4(2 - k_h^d)^2}, \quad \pi_i^d = 0,$$

$$\pi_w^d = m^d \sum_{h \in C} \Pi^d(h) \left\{ \frac{(k_w^d - 1) q}{4} + \frac{(1 - k_h^d) q}{2(2 - k_h^d)} \right\}.$$

which leads to the coalition worth  $v_p(C)$ :

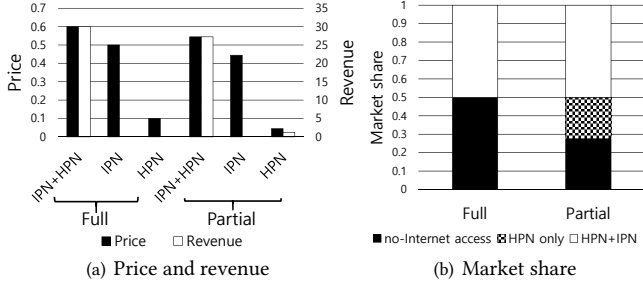
$$v_p(C) = \sum_{d \in \mathcal{D}} \frac{q m^d \sum_{h \in C} \Pi^d(h)}{4} \left\{ (k_w^d - 1) + \frac{(4 - k_h^d)(1 - k_h^d)}{(2 - k_h^d)^2} \right\}.$$

Note that for notational simplicity we omit ' $\star$ ' and the dependence on  $C$  in the notations, e.g.,  $p_h^d$  instead of  $p_h^{d,*}$  and  $k_h^d(C)$  instead of  $k_h^d$ .

Due to space limitation, we present the proof in our technical report [15]. The key technique is to use the standard backward induction.

**Interpretation.** We observe that the coalition of ISP and hosts under full cooperation tends to set higher prices, thus get larger worth, compared to the partial cooperation. This is due to the fact that in partial cooperation there is price competition between ISP-providing services (i.e., IPN-only and HPN+IPN) and hosts-providing service (i.e., HPN-only). Moreover, the coalition worths in both cooperation types increase with the cooperation size, because (i) the average expected QoS of HPN-only and HPN+IPN services (i.e.,  $k_h^d(C)$ )





**Figure 4: Prices, revenues and market shares at the equilibrium.** We choose  $|\mathcal{D}| = 1$ ,  $m^d = 100$  and the uniform mobility  $\Pi^d(j) = \frac{1}{H}$ ,  $k_h(j) = 0.2$  and  $k_w(j) = 1.2$  for all  $j \in \mathcal{H}$ .

and  $k_w^d(C)$  as well as (ii) the number of clients covered by  $C$  accordingly grow with the cooperation size.

## 4.2 Impact of Cooperation Type on Clients

**Market share.** Applying the equilibrium prices under both cooperation types in Theorem 4.1 to clients' service selection in (9), the following results on the market share  $\Theta_s^d$ , for each service are obtained: for each client mobility class  $d \in \mathcal{D}$ , and for a given coalition  $C$ ,

Type	no-Internet	HPN-only	IPN-only	HPN+IPN
Full	$[0, \frac{1}{2})$	$\emptyset$	$\emptyset$	$[\frac{1}{2}, 1]$
Partial	$[0, \frac{1}{2(2-k_h^d(C))})$	$[\frac{1}{2(2-k_h^d(C))}, \frac{1}{2})$	$\emptyset$	$[\frac{1}{2}, 1]$

This implies that the total market share in partial cooperation is larger than that in full cooperation. Interestingly, in full cooperation, only HPN+IPN service is operated, where the rest of the clients do not subscribe to any service. This is due to the fact that the equilibrium prices of HPN-only and IPN-only services are high compared to the clients' willingness to pay. For instance, if full cooperation reduces the prices, then the total revenue will decrease. That is why the equilibrium price is formed at the point that makes zero revenues from low-end services. However, in partial cooperation, the service by hosts, i.e., HPN-only, earns some profit from clients with low willingness to pay due to the capability of hosts' independent price selection. We also observe that under both cooperation types, no client subscribes to the IPN-only service at the equilibrium.

**Clients' utility.** Using the results in Theorem 4.1 for clients' utility functions in (4), we also obtain the following aggregate client utility:

$$\text{Full: } \sum_{d \in \mathcal{D}} m^d \sum_{h \in C} \Pi^d(h) \frac{k_w^d(C)q}{8}, \quad (13)$$

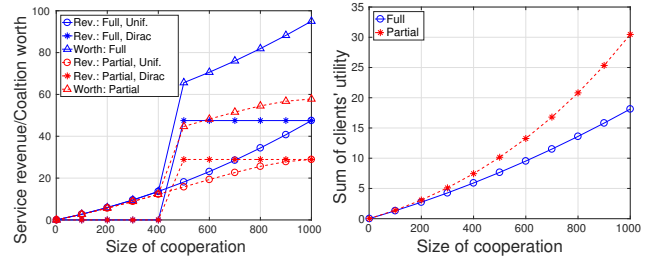
$$\text{Partial: } \sum_{d \in \mathcal{D}} m^d \sum_{h \in C} \Pi^d(h) \frac{q}{8} \left( k_w^d(C) + \frac{3k_h^d(C)}{2 - k_h^d(C)} \right). \quad (14)$$

Similarly to the equilibrium coalition worth, the aggregate utilities in both cooperation types also increase as the cooperation size, due to the increase of average service QoS,  $k_h^d(C)$  and  $k_w^d(C)$ . Moreover, it is easy to see larger aggregate client utility in partial cooperation than in the full cooperation, because the last term of the equation (14) is always positive with  $k_h^d \in [0, 1]$ . This is due

to the fact that the service prices in partial cooperation are lower than those in full cooperation, and the amount of the market share in partial cooperation gets larger, implying that a lower degree of cooperation is more beneficial to clients.

## 4.3 Numerical Examples

We now show numerical examples of a simple homogeneous setup to present more implications of our analytical results. We consider a single mobility class, i.e.,  $|\mathcal{D}| = 1$ , where such class defines the uniform mobility pattern that every host is visited with equal probability, i.e.,  $\Pi^d(j) = \frac{1}{H}$ , for all  $j \in \mathcal{H}$ . We consider a homogeneous average QoS for each service with  $k_h(j) = 0.2$  and  $k_w(j) = 1.2$  for all  $j \in \mathcal{H}$ . We choose other parameters as:  $m^d = 100$  and  $H = 1000$ . Figure 4(a) shows that ISP and hosts in full cooperation indeed set higher prices and that the revenue of IPN-only service is zero. This implies that a higher degree of cooperation such as full cooperation results in a higher equilibrium price, because they, as a coalition, have large market power against clients. However, the total market share of services in partial cooperation is larger than that in full cooperation by about 50%, as seen in Figure 4(b).



(a) Impact of cooperation type and clients' mobility on revenue and worth. (b) Impact of cooperation type on clients' utility.

**Figure 5: Revenue, worth and clients' utility.** We choose  $k_h(j) = 0.9$  and  $k_w(j) = 1.9$  for all  $j \in \mathcal{H}$  as well as  $|\mathcal{D}| = 2$  in Figure 5(a), and  $|\mathcal{D}| = 1$  in Figure 5(b), where  $m^d = 100$  for all  $d \in \mathcal{D}$ .

Figure 5 shows the coalition worth and the aggregate client utility at the equilibrium for both cooperation types, where we choose  $k_h(j) = 0.9$  and  $k_w(j) = 1.9$  for all  $j \in \mathcal{H}$ . In Figure 5(a), we consider two mobility classes, i.e.,  $|\mathcal{D}| = 2$ , where one is uniform distribution and the other is Dirac delta distribution to see two extreme mobility patterns. In the uniform distribution,  $\Pi^{\text{Unif.}}(j) = \frac{1}{H}$ , for all  $j \in \mathcal{H}$ , and in the Dirac delta distribution,  $\Pi^{\text{Dirac}}(j) = 1$  if  $j = h$ , and  $\Pi^{\text{Dirac}}(j) = 0$ , otherwise. We observe that the worth only increases, whenever the host  $h \in \mathcal{H}$  with  $\Pi^{\text{Dirac}}(j) > 0$  joins in the coalition. However, the worth increases with the cooperation size, if clients uniformly visit all HPNs. Moreover, we see that the worth is convex with respect to the size of full cooperation, while it is concave with respect to the size of partial cooperation. As seen in Figure 5(b), partial cooperation always guarantees a higher aggregate utility than full cooperation.

## 5 INCENTIVE FOR COOPERATION: FLUID SHAPLEY VALUE AND STABILITY

### 5.1 Fluid Worth and Fluid Shapley Value

In Section 4, we derived the form of the worth functions  $v_f(C)$  and  $v_p(C)$  for a given coalition  $C$  under full and partial cooperations, respectively. This defines two coalitional games  $(\mathcal{G}, v_f)$  and  $(\mathcal{G}, v_p)$ . We now study how to share the worth among the players in  $C$ , for which we consider the famous Shapley value. Due to the combinatorial structure of Shapley value as characterized in (3), it is notoriously challenging to analytically characterize or even numerically compute it when the size of coalition is large (i.e., NP-hard). This motivates us to consider an approximation approach, the fluid-limit of Shapley value (or simply fluid Shapley value), as defined in what follows:

*Definition 5.1 (Fluid Shapley Value).* For a given coalition  $C$ , let  $\phi^i(C)$  and  $\phi^h(C)$  be the Shapley values of the ISP and host  $h$  in  $C$ . Then, we define their fluid Shapley values  $\bar{\phi}^i(C)$  and  $\bar{\phi}^h(C)$  as:

$$\bar{\phi}^i(C) \triangleq \lim_{H \rightarrow \infty} \frac{\phi^i(C)}{H}, \quad \bar{\phi}^h(C) \triangleq \lim_{H \rightarrow \infty} \phi^h(C), \quad \forall h \in \mathcal{H}. \quad (15)$$

Note that the number of clients also scales as  $H$  with its ratio of  $\alpha$ , where we define that  $\alpha^d = m^d/H$ . Also, we note that the ISP's fluid Shapley value  $\bar{\phi}^i(C)$  is the per-host average value. This is because when the number of clients scales, the market size and ISP's investment would also scale (to maintain a certain fixed level of IPN's QoS, i.e.,  $q$ ), and thus, ISP's entire revenue accordingly scales as  $H$ .

We often trade model's simplicity for analytical tractability in theoretical research, because we believe that the results even with simple assumptions provide useful messages and practical implications. In this paper, we make the following assumptions on the division of hosts into certain *classes*, where the hosts in the same class are homogeneous, thus their Shapley value are equivalent inside each class (but, still allowing heterogeneity for different classes).

**A1. Host class:** We define  $\mathcal{L} = (1, \dots, L)$  as the set of host classes, where hosts in the same class have the same QoS. Let  $H_l$  be the number of hosts in class  $l$ , thus  $\sum_{l \in \mathcal{L}} H_l = H$ . Thus, we use the notations of QoS for HPN+IPN and HPN-only of  $k_w(l)$  and  $k_h(l)$  for class  $l \in \mathcal{L}$ . Let  $\mathbf{k}_w = [k_w(l)]_{l \in \mathcal{L}}$ ,  $\mathbf{k}_h = [k_h(l)]_{l \in \mathcal{L}}$ .

**A2. Clients' mobility:** We assume that clients stay at the hosts in the same class uniformly at random. Thus, by letting  $[\Pi^d(l)]_{l \in \mathcal{L}}$  the probability that a class  $d$  client stays at the hosts of class  $l$ , we have  $\Pi^d(l) = H_l \times \Pi^d(h)$ . Let  $\Pi^d = [\Pi^d(l)]_{l \in \mathcal{L}}$ .

**Fluid worth and fluid Shapley value.** To derive the fluid Shapley value, we first derive the fluid worth function,  $\bar{v}_t(\mathbf{x}) := \lim_{H \rightarrow \infty} \frac{v_t(\mathbf{x})}{H}$ ,  $t \in \{f, p\}$ , where  $\mathbf{x} = \mathbf{x}(C) = [\frac{H_l(C)}{H}]_{l \in \mathcal{L}}$ , and  $H_l(C)$  is the number of class- $l$  hosts that are included only in coalition  $C$ . Then, Lemma 5.2 derives a closed form of the fluid worth function.

**LEMMA 5.2 (FLUID WORTH).** *The fluid worth functions under full and partial cooperations are characterized as: for a given coalition  $C$ ,*

- Full cooperation:

$$\bar{v}_f(\mathbf{x}) = \frac{q}{4} \boldsymbol{\alpha}^T \text{diag}(\boldsymbol{\rho})(\boldsymbol{\kappa}_w - \boldsymbol{\rho} + 1), \quad (16)$$

where  $\boldsymbol{\rho} = [\rho^d]_{d \in \mathcal{D}}$  and  $\boldsymbol{\kappa}_w = [\kappa_w^d]_{d \in \mathcal{D}}$  are defined by:

$$\rho^d = \mathbf{g}^T \text{diag}(\Pi^d) \mathbf{x}, \quad \kappa_w^d = \mathbf{k}_w^T \text{diag}(\Pi^d) \text{diag}(\mathbf{g}) \mathbf{x},$$

where  $\mathbf{g} = [H/H_l]_{l \in \mathcal{L}}$ .

- Partial cooperation:

$$\bar{v}_p(\mathbf{x}) = \frac{q}{4} \boldsymbol{\alpha}^T \text{diag}(\boldsymbol{\rho}) \boldsymbol{\lambda}, \quad (17)$$

where  $\boldsymbol{\lambda} = [\lambda^d]_{d \in \mathcal{D}}$  and  $\kappa_h^d$  are defined by:

$$\lambda^d = (\kappa_w^d - \rho^d) + \frac{(4 - \kappa_h^d)(1 - \kappa_h^d)}{(2 - \kappa_h^d)^2}, \quad \kappa_h^d = \mathbf{k}_h^T \text{diag}(\Pi^d) \text{diag}(\mathbf{g}) \mathbf{x},$$

where  $\mathbf{g}$  is the same as the one in full cooperation.

**PROOF.** We first derive  $\bar{v}_f(\mathbf{x})$ . By the definition of  $k_w^d(C)$  shown in (1), we get,  $k_w^d(C) = \sum_{j \in C \setminus \{I\}} \Pi^d(j)(k_w(j) - 1) + 1$ . Using the assumptions **A1** and **A2**,  $k_w^d(C)$  can be rewritten by:  $k_w^d(C) = \sum_{l \in \mathcal{L}} x_l g_l k_w(l) \Pi^d(l) - \sum_{l \in \mathcal{L}} x_l g_l \Pi^d(l) + 1$ . Similarly,  $m^d(C)$  can be rewritten by:  $m^d(C) = \sum_{h \in C} \Pi^d(h) = m^d \sum_{l \in \mathcal{L}} \Pi^d(l) g_l x_l$ . Therefore,  $v_f(\mathbf{x}(C))$  is given by:

$$\begin{aligned} \lim_{H \rightarrow \infty} \frac{v_f(\mathbf{x}(C))}{H} &= \sum_{d \in \mathcal{D}} \frac{q \alpha^d}{4} \left( \sum_{l \in \mathcal{L}} \Pi^d(l) g_l x_l \right) \\ &\quad \times \left( \sum_{l \in \mathcal{L}} x_l g_l k_w(l) \Pi^d(l) - \sum_{l \in \mathcal{L}} x_l g_l \Pi^d(l) + 1 \right), \end{aligned}$$

where  $\alpha^d = m^d/H$ . Finally, we have,

$$\bar{v}_f(\mathbf{x}) = \lim_{H \rightarrow \infty} \frac{v_f(\mathbf{x}(C))}{H} = \frac{q}{4} \boldsymbol{\alpha}^T \text{diag}(\boldsymbol{\rho})(\boldsymbol{\kappa}_w - \boldsymbol{\rho} + 1),$$

where  $\boldsymbol{\rho} = [\rho^d]_{d \in \mathcal{D}}$  and  $\boldsymbol{\kappa}_w = [\kappa_w^d]_{d \in \mathcal{D}}$  are defined in (16). Similarly to  $\bar{v}_f(\mathbf{x})$ , we can derive  $\bar{v}_p(\mathbf{x})$ .  $\square$

It is easy to check that the fluid worth function is partially differentiable with respect to  $\mathbf{x}$ . Using this and from Lemma 5.2, we now state our main result on the analytical form of the fluid Shapley value under both cooperation types.

**THEOREM 5.3 (FLUID SHAPLEY VALUE).** *Under each cooperation type  $t \in \{f, p\}$ , for a given coalition  $C$ , the fluid Shapley values  $\bar{\phi}_t^i(\mathbf{x}(C))$  and  $\bar{\phi}_t^l(\mathbf{x}(C))$  of ISP and class- $l$  hosts have the following closed form:*

- Full cooperation: For all  $\mathbf{x} > \mathbf{0}$ ,

$$\bar{\phi}_f^i(\mathbf{x}) = \frac{q}{12} \boldsymbol{\alpha}^T \text{diag}(\boldsymbol{\rho})(\boldsymbol{\kappa}_w - \boldsymbol{\rho}) + \frac{q}{8} \boldsymbol{\alpha}^T \boldsymbol{\rho},$$

$$\bar{\phi}_f^l(\mathbf{x}) = \sum_{d \in \mathcal{D}} \frac{\alpha^d q}{12} g_l \Pi^d(l) \left( (k_w(l) - 1) \rho^d + \kappa_w^d - \rho^d + \frac{3}{2} \right), \quad \forall l \in \mathcal{L}.$$

- Partial cooperation: For all  $\mathbf{x} > \mathbf{0}$ ,

$$\begin{aligned} \bar{\phi}_p^i(\mathbf{x}) &= \frac{q}{4} \sum_{d \in \mathcal{D}} \alpha^d \rho^d \left( \frac{1}{3} (\kappa_w^d - \rho^d) + \frac{4}{(\kappa_h^d)^2 (\kappa_h^d - 2)} \right. \\ &\quad \left. - \frac{4 \log\left(\frac{2 - \kappa_h^d}{2}\right) - 2}{(\kappa_h^d)^2} - \frac{1}{\kappa_h^d} + \frac{1}{2} \right), \end{aligned}$$

$$\bar{\phi}_p^l(\mathbf{x}) = \frac{q}{4} \sum_{d \in \mathcal{D}} \alpha^d g_l \Pi^d(l) \left( \frac{2}{3} (k_w(l) - 1) \rho^d + \frac{1}{2} \right)$$

$$+ \frac{8 - 4(\kappa_h^d)^2}{(\kappa_h^d)^2(\kappa_h^d - 2)^2} + \frac{4 \log(\frac{2 - \kappa_h^d}{2}) - 2}{(\kappa_h^d)^2}, \forall l \in \mathcal{L}. \quad (18)$$

PROOF. As the first step, we derive the fluid Shapley value so as to satisfy the axioms of the Shapley value. From the axiom of efficiency, i.e.,  $\sum_{j \in C} \varphi^j(C) = v(C)$ , we get  $\varphi_t^i(C) + \sum_{l \in \mathcal{L}} H_l(C) \cdot \varphi_t^l(C) = v_t(C)$ ,  $\forall C$  for type- $t$  cooperation,  $t = \{p, f\}$ . Multiplying both sides by  $1/H$  and taking limits  $H \rightarrow \infty$ , we have

$$\bar{\varphi}_t^i(\mathbf{x}) + \sum_{l \in \mathcal{L}} x_l \cdot \bar{\varphi}_t^l(\mathbf{x}) = \bar{v}_t(\mathbf{x}), \forall \mathbf{x}. \quad (19)$$

Also, from the axiom of fairness/balanced contribution, i.e., for any  $j, k \in C$ ,  $\varphi^j(C) - \varphi^j(C \setminus \{k\}) = \varphi^k(C) - \varphi^k(C \setminus \{j\})$ , we have  $\varphi_t^i(C) = \varphi_t^i(C) - \varphi_t^i(C \setminus \{I\})$ ,  $\forall C$ , where  $v_t(C \setminus \{I\}) = 0$ . Multiplying both sides by  $1/H$  and taking limits  $H \rightarrow \infty$ , we have,

$$\bar{\varphi}_t^l(\mathbf{x}) = \frac{\partial \bar{\varphi}_t^i(\mathbf{x})}{\partial x_l}, \forall \mathbf{x}. \quad (20)$$

From (19) and (20), we get a differential equation as:

$$\bar{\varphi}_t^i(\mathbf{x}) + \sum_{l \in \mathcal{L}} x_l \cdot \frac{\partial \bar{\varphi}_t^i(\mathbf{x})}{\partial x_l} = \bar{v}_t(\mathbf{x}), \forall \mathbf{x}. \quad (21)$$

Then, the solution of (21) is the fluid Shapley value of ISP under the  $t$ -type cooperation, which is given by:

$$\bar{\varphi}_t^i(\mathbf{x}) = \int_0^1 \bar{v}_t(u\mathbf{x}) du. \quad (\text{Fluid SV of ISP}) \quad (22)$$

Moreover, from (20), the fluid Shapley value of hosts in level  $l$  under the  $t$ -type cooperation is given by:

$$\bar{\varphi}_t^l(\mathbf{x}) = \int_0^1 u \cdot \frac{\partial \bar{v}_t}{\partial x_l}(u\mathbf{x}) du, \forall l \in \mathcal{L}. \quad (\text{Fluid SV of host}) \quad (23)$$

As the second step, we calculate the explicit form of fluid Shapley values. For example, from (22) and (16), we get the fluid Shapley value of ISP under the full cooperation as below:

$$\begin{aligned} \bar{\varphi}_f^i(\mathbf{x}) &= \int_0^1 \bar{v}_f(u\mathbf{x}) du = \int_0^1 \sum_{d \in \mathcal{D}} \frac{q\alpha^d}{4} \left( \sum_{l \in \mathcal{L}} \Pi^d(l) g_l u x_l \right) \\ &\quad \times \left( \sum_{l \in \mathcal{L}} u x_l g_l k_w(l) \Pi^d(l) - \sum_{l \in \mathcal{L}} u x_l g_l \Pi^d(l) + 1 \right) du \\ &= \sum_{d \in \mathcal{D}} \frac{q\alpha^d}{4} \int_0^1 (A u^2 + B u) du = \sum_{d \in \mathcal{D}} \frac{q\alpha^d}{4} \left( \frac{A}{3} + \frac{B}{2} \right), \end{aligned}$$

where  $A$  and  $B$  are defined by:

$$B = \sum_{l \in \mathcal{L}} x_l g_l \Pi^d(l), \quad A = \left( \sum_{l \in \mathcal{L}} (k_w(l) - 1) x_l g_l \Pi^d(l) \right) \times B.$$

Thus, we have  $\bar{\varphi}_f^i(\mathbf{x}) = \frac{q}{12} \alpha^T \text{diag}(\rho)(\kappa_w - \rho) + \frac{q}{8} \alpha^T \rho$ . Similarly to  $\bar{\varphi}_f^i(\mathbf{x})$ , we can derive  $\bar{\varphi}_f^j(\mathbf{x})$ ,  $\bar{\varphi}_p^i(\mathbf{x})$  and  $\bar{\varphi}_p^l(\mathbf{x})$ .  $\square$

To the best of our knowledge, characterizing a Shapley value as a closed form (even as its fluid-limit) is far from being straightforward, especially when the worth function is not given as a specific form, but comes from an equilibrium of a non-trivial embedded non-cooperative dynamic game as in this paper. In each coalition, ISP is a crucial player (i.e., a veto player) without whom no worth is generated. This is why we consider a per-host fluid Shapley value for the ISP, as defined in (15). We provide more interpretations of Theorem 5.3 using numerical examples in Section 5.3. In the next section, we investigate when ISP and hosts have incentives to form

a coalition or to maintain the grand coalition, often referred to as *stability* under the revenue sharing based on the Shapley value.

## 5.2 Stability of UPN

In this section, we establish our results on the stability of the coalition of ISP and hosts, under our revenue sharing rule, Shapley value. Even if the worth under a given coalition  $C$  is fairly divided to each player, it does not straightforwardly guarantee coalition  $C$ 's stability whose formal definition will be introduced shortly. Thus, we aim at understanding how the Shapley value impacts on the stability of a given coalition under both cooperation types. Of particular interest is the stability of the grand coalition  $\mathcal{G}$  in this paper.

**Background.** Under the assumption that we use Shapley value as a revenue sharing rule, we consider two popular notions of stability [25] in this paper: (i) individual-rationality and (ii) core-stability as defined in what follows:

*Definition 5.4.* Consider a coalitional game  $(\mathcal{G}, v)$ . Shapley value is said to be *individual-rational* if  $\varphi^j(C) \geq v(\{j\})$ ,  $\forall j \in C$ ,  $\forall C \subseteq \mathcal{G}$ .

*Definition 5.5.* Consider a coalitional game  $(\mathcal{G}, v)$ . Shapley value lies in the *core* if  $\sum_{j \in \mathcal{G}} \varphi^j(\mathcal{G}) = v(\mathcal{G})$  and  $\sum_{j \in C} \varphi^j(\mathcal{G}) \geq v(C)$ ,  $\forall C \subseteq \mathcal{G}$ .

In Definitions 5.4 and 5.5, if the conditions hold, we also say that the grand coalition  $\mathcal{G}$  is *individually-stable* and *core-stable* (under Shapley value), respectively. In individual-rationality, each player does not have incentive to individually deviate from the grand coalition. Core-stability is a stronger one, meaning that there exists no subcoalition  $C$  of the players whose aggregate worth exceeds that of the grand coalition  $\mathcal{G}$ . For example, if a coalition excluding a player  $j$ , i.e.,  $\mathcal{G} \setminus \{j\}$ , gains a larger worth than the aggregated Shapley values of the players in  $\mathcal{G} \setminus \{j\}$ , then the grand coalition will be broken, since the coalition excluding the player  $j$  will kick the unprofitable player  $j$  out. In this case, even though Shapley value under the grand coalition is individually rational for all players including  $j$  but it is not core-stable due to unprofitable  $j$ .

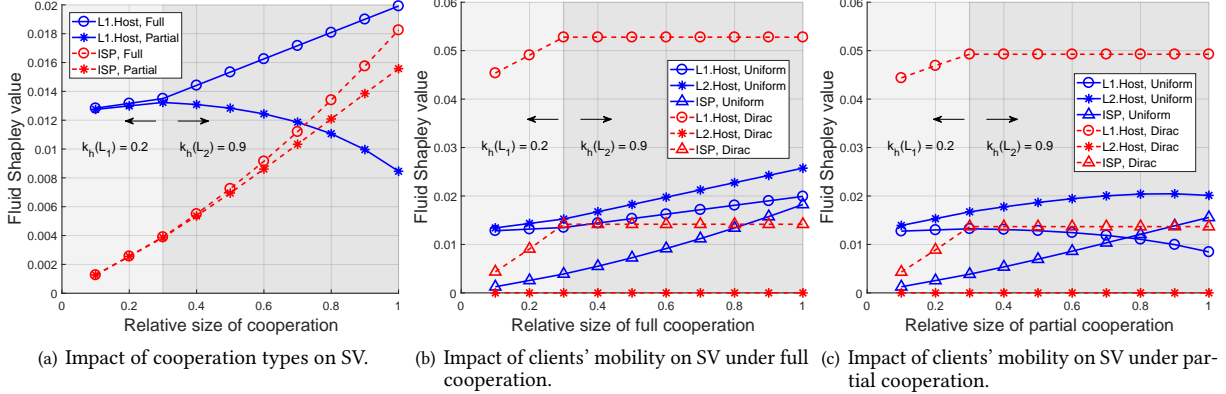
**Stability of UPN Market.** We now present Theorem 5.6 on the stability in terms of individual-rationality and core-stability.

**THEOREM 5.6 (STABILITY).** For the coalitional games  $(\mathcal{G}, v_f)$  and  $(\mathcal{G}, v_p)$ , the following holds: Under the assumptions **A1** and **A2** and in the fluid-limit regime when  $H \rightarrow \infty$ , as in (15),

- (i) Shapley value is individual-rational for both  $(\mathcal{G}, v_f)$  and  $(\mathcal{G}, v_p)$ .
- (ii) Shapley value lies in the core for  $(\mathcal{G}, v_f)$ .

PROOF. (i) It suffices to show that the worth functions  $v_t$  for all  $t \in \{f, p\}$  are superadditive [25], i.e., for all  $C_1, C_2 \subseteq C$  such that  $C_1 \cap C_2 = \emptyset$ , we have  $v_t(C_1 \cup C_2) \geq v_t(C_1) + v_t(C_2)$ . Suppose that  $C_1$  contains the ISP. Then,  $v_t(C_2) = 0$ , thus we only need to show that  $v_t(C_1 \cup C_2) \geq v_t(C_1)$ . Then, the above inequality is true since the following holds:  $\frac{\partial \bar{v}_t(\mathbf{x})}{\partial x_l} \geq 0$ ,  $\forall l \in \mathcal{L}$ ,  $\forall t \in \{f, p\}$ . Therefore, the Shapley value of games  $(\mathcal{G}, v_t)$  for all  $t \in \{f, p\}$  are individual-rational. (ii) Similarly, it suffices to show that the worth function  $v_f$  is supermodular [25], i.e., for all  $C_1 \subseteq C_2 \subseteq C \setminus \{j\}$ ,  $\forall j \in C$ ,  $v_f(C_1 \cup \{j\}) - v_f(C_1) \leq v_f(C_2 \cup \{j\}) - v_f(C_2)$ . Note that  $\bar{v}_f : \mathbb{R}^L \mapsto \mathbb{R}$  is twice differentiable. Then, the supermodularity holds if the second derivative  $\frac{\partial^2 \bar{v}_f(\mathbf{x})}{\partial x_l \partial x_r}$  is nonnegative for all distinct





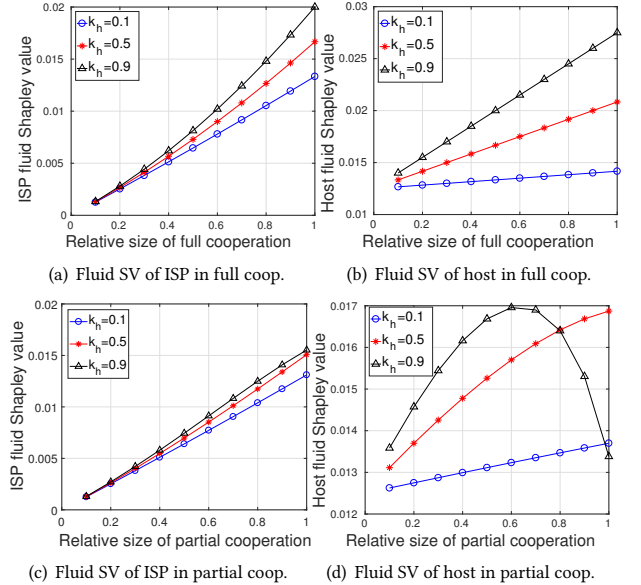
**Figure 6: Fluid Shapley values.**  $|\mathcal{L}| = 2$ ,  $k_h(L_1) = 0.2$ ,  $k_h(L_2) = 0.9$ ,  $H_{L_1} = 300$ ,  $H_{L_2} = 700$ ,  $|\mathcal{D}| = 1$ , and  $m^d = 100$ ,  $\forall d \in \mathcal{D}$ .

indexes  $l \in \mathcal{L}$  and  $r \in \mathcal{L}$  and for any feasible  $\mathbf{x} \in \mathbb{R}^L$ . Recall that  $\bar{v}_f(\mathbf{x})$  is given by:  $\bar{v}_f(\mathbf{x}) = \frac{1}{4} \alpha^T \text{diag}(\rho)(\kappa_w - \rho + 1)$ , whose second derivative satisfies:  $\frac{\partial^2 \bar{v}_f(\mathbf{x})}{\partial x_l \partial x_r} \geq 0$ ,  $\forall l, r \in \mathcal{L}$ . Thus, the Shapley value of the game  $(\mathcal{G}, v_f)$  lies in the core.  $\square$

In terms of individual-rationality, Theorem 5.6 implies that every individual player does not have incentive to deviate from the grand coalition irrespective of whether ISP and hosts cooperate fully or partially, when the given revenue sharing rule is fair enough based on Shapley value. This result is somewhat meaningful for ISP, since it convinces the ISP of the individual gain by participating in any type UPN market. Furthermore, for core-stability, Theorem 5.6 also indicates that any group of players *does not break* the grand coalition to be better-off under full cooperation with Shapley value revenue sharing rule. In general, it is extremely hard to prove the core-stability of a revenue sharing rule due to the computational complexity. However, the Shapley value based revenue sharing has a nice property that if the worth function satisfies the convexity, then the core-stability is induced. Consequently, under the full cooperated UPN market, ISP and hosts have no incentive to kick any group of players out with Shapley value based revenue sharing. In numerical examples, we will shortly present the case when core-stability is not maintained, when ISP and hosts partially cooperate. It will be an interesting future work to analytically characterize the conditions under which core-stability is ensured for partial cooperation.

### 5.3 Numerical Results: Fluid SV and Stability

In this section, we illustrate numerical examples to discuss the implications of fluid Shapley value and stability, which are analytically developed in the previous section. Figure 6 depicts the fluid Shapley values with respect to the relative size of cooperation, i.e.,  $\frac{\sum_{l \in \mathcal{L}} H_l(C)}{H}$ , when  $H = 1000$ . In Figure 6, we consider two host classes that are differentiated by their provided QoS, i.e.,  $\mathcal{L} = \{L_1, L_2\}$ , where the average QoS and the numbers of hosts are:  $k_h(L_1) = 0.2$ ,  $k_h(L_2) = 0.9$ ,  $k_w = 1 + k_h$ ,  $H_{L_1} = 300$ , and  $H_{L_2} = 700$ . We choose  $|\mathcal{D}| = 1$  in terms of clients' mobility class and  $m^d = 100$  where we consider two scenarios of mobility pattern: (i) the Dirac delta mobility pattern:  $\Pi^{\text{Dirac}}(j) = \frac{1}{H_{L_1}}$ ,  $\forall j \in \mathcal{L}_1$ , and  $\Pi^{\text{Dirac}}(j) = 0$ ,  $\forall j \in \mathcal{L}_2$ , and (ii) the uniform mobility pattern:  $\Pi^{\text{Uniform}}(j) = \frac{1}{H}$ ,  $\forall j \in \mathcal{H}$ .



**Figure 7: Fluid Shapley value with different HPN-only QoS,  $k_h$ .** We choose  $|\mathcal{L}| = 1$ ,  $H = 1000$ ,  $|\mathcal{D}| = 1$ , and  $m^d = 100$ .

Figure 6(a) shows that the fluid Shapley values in different cooperation types. In full cooperation, the fluid Shapley values of ISP and hosts increase as the number of hosts who participate in the coalition grows. This indirectly demonstrates the stability of Shapley value in full cooperation, since it indicates the largest fluid Shapley value in the largest cooperation. However, Figure 6(a) also shows that the fluid Shapley value in partial cooperation tends to decrease with the cooperation size. It implies that hosts may not be willing to agree to a grand coalition when cooperation tie becomes loose, thus instability of the grand coalition. Figures 6(b) and 6(c) show the impact of clients' mobility patterns on the fluid Shapley value under both cooperations. In each cooperation, when the clients' mobility pattern follows the Dirac delta distribution, the fluid Shapley value of L1-class hosts increases as the number of incoming L1-class hosts grows. However, it immediately stops as a response to the injection of L2-class hosts. In order to show such a phenomenon more clearly, we generate a scenario that all L1-class hosts join in the cooperation first and then L2-class hosts follows.

Also, when clients' mobility pattern is uniform, the fluid Shapley values of ISP and hosts in full cooperation always increase as hosts of any class enter the cooperation, as seen in Figure 6(b). However, Figure 6(c) shows that the fluid Shapley values of hosts in partial cooperation have a decreasing tendency.

In Figure 7, we illustrate the fluid Shapley values  $k_h$  and  $k_w$  with varying service QoS of HPN-only and HPN+IPN. We consider  $|\mathcal{D}| = 1$ , where such a class defines the uniform mobility pattern. Moreover, we assume that a single host level, i.e.,  $|\mathcal{L}| = 1$ , and choose  $H = 1000$ . In this case, we numerically show the fluid Shapley values with three different  $k_h = 0.1, 0.5, 0.9$ . with  $k_w = 1 + k_h$ . Figures 7(a) and 7(b) (fluid Shapley values under full cooperation) and Figure 7(c) (ISP's fluid Shapley value under partial cooperation) show a higher fluid Shapley value for a higher QoS, while Figure 7(d) (hosts' fluid Shapley value under partial) does not. It implies that a high QoS of HPN-only service, i.e.,  $k_h = 0.9$  discourages the core-stability of the grand coalition in partial cooperation, because under partial cooperation when the HPN's QoS is high and the cooperation size is large, hosts' contribution to the cooperation is not significant due to the price competition between ISP and hosts.

## 6 CONCLUSION AND LIMITATION

**Conclusion.** We analyzed a market of UPN, consisting of ISP, hosts, and clients via a mixture of cooperative and non-cooperative games, where we model various heterogeneities in willingness to pay and mobility pattern of clients, hosts' QoS, and type of cooperation among ISP and hosts. Our analysis provides (i) when and how three types of players are economically beneficial and (ii) when they are willing to stay in their cooperation. The key technical ingredient is the inter-play between the strategic behaviors of ISP/hosts/clients, modeled by an embedded non-cooperative dynamic game and a coalitional game to study how to distributed the total worth to ISP and hosts. We consider Shapley value based revenues sharing and studied its stability for partial and full cooperations, thanks to tractability via its fluid-limit approximation.

**Limitations.** Despite the hardness in characterizing Shapley value and its stability, our analysis is able to capture a certain degree of heterogeneity of clients, hosts and ISP. However, we still made a few assumptions for analytical tractability, which is clearly the limitation of this paper, yet inevitable to obtain a set of theory-inspired conclusions. To summarize, first, we assumed a single ISP, thus we could not analyze the competitive or cooperative behaviors among multiple ISPs. Also, to simplify the interaction between providers (ISP and hosts) and customers (clients), we assume the logically separated role of end-users as hosts and clients. However, in practice, a host may also be a client, and such a dual role may change how the market forms. Third, only class-based heterogeneity in terms of mobility pattern, thus being homogeneous inside a class, was assumed in the analysis of Shapley value and its stability (but not assumed in the characterization of worth function). However, this class-based approach for modeling heterogeneity is quite common for the case of a large number of populations, to the best of our knowledge. Finally, we did not model the change of QoS, depending on the number of subscribers in IPN and HPN, where we mainly focused on the case where the available resources in both networks are not significantly scarce, with more emphasis on the macroscopic

features of a UPN market. Clearly, studying with removing all or some of these limitations is left as a good future work.

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